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$\frac{1}{16}$-BPS black holes and giant gravitons in the $\mathrm{AdS}_5 \times \mathrm{S}^5$ Space

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Abstract: We explore $\frac{1}{16}$-BPS objects of type IIB string theory in $AdS_5 \times S^5$. First, we consider supersymmetric AdS_5 black holes, which should be $\frac{1}{16}$-BPS and have a characteristic that not all physical charges are independent. We point out that the Bekenstein-Hawking entropy of these black holes admits a remarkably simple expression in terms of (dependent) physical charges, which suggests its microscopic origin via certain Cardy or Hardy-Ramanujan formula. We also note that there is an upper bound for the angular momenta given by the electric charges. Second, we construct a class of $\frac{1}{16}$-BPS giant graviton solutions in $AdS_5 \times S^5$ and explore their properties. The solutions are given by the intersections of $AdS_5 \times S^5$ and complex 3 dimensional holomorphic hyperspaces in $\mathbb{C}^{1+5}$, the latter being the zero loci of three holomorphic functions which are homogeneous with suitable weights on coordinates. We investigate examples of giant gravitons, including their degenerations to tensionless strings.

Keywords: AdS-CFT Correspondence, Black Holes in String Theory, D-branes.

Contents

1. Introduction

The type IIB superstring theory on $AdS_5 \times S^5$ contains a large class of BPS states in its spectrum. In the $\frac{1}{2}$-BPS sector, the BPS states carry nonzero $\mathrm{U}(1) \subset \mathrm{SO}(6)$ momenta or the R-charges and have been extensively studied, both in the string theory and the $\mathcal{N} = 4$ Yang-Mills theory. When this R-charge is large, the gravitons carrying this charge expand into spherical $D3$ branes in S^5 or AdS_5, which is called the giant gravitons [1 − 5]. More recently some pioneering works in the supergravity and the super Yang-Mills theory have been done in [6, 7], as well as many others. Just to mention a few of them, [8 − 12].

Down to $\frac{1}{8}$-BPS sector, BPS states carrying three R-charges in $\mathrm{U}(1)^3 \subset \mathrm{SO}(6)$ have also been studied. Especially, in the probe limit, the classical configurations are given by $D3$ branes expanding in S^5 with their shapes given by holomorphic surfaces in $\mathbb{C}^3$ [13]. See also [14 − 18] for related works in the $\frac{1}{8}$-BPS sector. The same sector has also been investigated from the supergravity viewpoint. Supergravity solutions carrying $\mathrm{U}(1)^3$ charges in the internal S^5 are called superstars [19, 20], which develop naked singularities. These solutions are interpreted in [19] as distributions of giant gravitons.

It would be interesting to consider the even less supersymmetric sector, the $\frac{1}{16}$-BPS one preserving 2 real supersymmetries. One motivation for this comes from the study of supersymmetric AdS_5 black holes, which have been obtained rather recently [21 − 25] from 5 dimensional gauged supergravity theories. These black holes carry three $\mathrm{U}(1)^3 \subset \mathrm{SO}(6)$ momenta in S^5 as well as two $\mathrm{U}(1)^2 \subset \mathrm{SO}(4)$ in AdS_5. Note that, in order to have black holes with regular horizons, the angular momenta in AdS_5 should be nonzero: otherwise the solutions would develop naked singularities [19, 20]. The presence of angular momenta

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in AdS_5 forces them to preserve no larger than $\frac{1}{16}$ supersymmetry. To understand these objects, it will also be interesting to consider the microscopic side of this sector and try to compare it with the supersymmetric black holes. The purpose of this paper is to explore the macroscopic and microscopic aspects of the $\frac{1}{16}$-BPS objects, hoping that our results would provide important clues for future works toward relating both sides: for instance, counting the black hole microstates. For the previous attempts to understand microscopic aspects of these black holes, see [26, 27]. (See [28] also.)

From the macroscopic side, we consider the Bekenstein-Hawking entropy of supersymmetric AdS_5 black holes. As we already mentioned, they carry two angular momenta in AdS_5, call it J_1 and J_2, as well as three $\mathrm{U}(1)^3 \subset \mathrm{SO}(6)$ charges Q_I $(I = 1, 2, 3)$. Another odd feature of the supersymmetric AdS black holes (known to date) is that these physical charges are not all independent in order to have regular black hole solutions.[1] Therefore, there should be an ambiguity if one tries to write down the expression for the entropy in terms of (dependent) physical charges, which one usually does in order to compare it to the microscopic one. Indeed, in the literatures, implicit prescriptions are made on this ambiguity [26, 27] in special regimes of charges. We observe that, in terms of dependent physical charges, the entropy admits the following simple expression,

$$S = 2\pi\sqrt{Q_1Q_2 + Q_2Q_3 + Q_3Q_1 - N^2 J}\,, \tag{1.1}$$

where $J = \frac{J_1+J_2}{2}$ is the self-dual part of the angular momentum. (See section 2 for the details and generalizations.) Compared to the expressions advocated in previous works, this expression is exact in all regime of charges. The factor N^2 originates from the central charge of the $\mathcal{N} = 4$ Yang-Mills theory which is holographically dual to the string theory in $AdS_5 \times S^5$: $4c = N^2$. We think the expression is remarkably simple that there should be a nice microscopic explanation, for instance, via a Cardy or Hardy-Ramanujan formula of a certain microscopic model.

The above observation would be a motivation for investigating the $\frac{1}{16}$-BPS sector microscopically. In this diretion, we directly generalize the work of [13] to construct $\frac{1}{16}$-BPS $D3$ giant graviton solutions in the probe limit and study their properties. Turning off the worldvolume gauge fields, we find that $\frac{1}{16}$-BPS brane embeddings and time evolutions are given by the intersectionss of holomorphic 3-manifolds in $\mathbb{C}^{1+2} \times \mathbb{C}^3$ with $AdS_5 \times S^5$. With a suitable choice of complex structure of $\mathbb{C}^{1+2} \times \mathbb{C}^3$ with coordinates Y^A $(A = 1, \cdots, 6)$, the 3-manifold is given by the zero locus of three holomorphic functions $F_k(Z^A) = 0$ $(k = 1, 2, 3)$, where the functions should be homogeneous in the following sense,

$$F_k(\lambda Y^0, \lambda Y^1, \lambda Y^2, \lambda^{-1}Y^3, \lambda^{-1}Y^4, \lambda^{-1}Y^5) = \lambda^{d_k} F(Y^A) \tag{1.2}$$

with suitable degrees d_k. It will be interesting to construct explicit examples which are relevant to the black hole physics. We do not have much to say about it so far, except for a few comments in section 5.

[1] See [21] for some arguments on this issue for black holes in minimal gauged supergravity, and [29, 30] for examples of supersymmetric AdS_7 and AdS_4 black holes with the same property. We are not sure whether this phenomenon is an essential or a technical one. However, see [27].

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As an aside, we use these giant graviton solutions to illustrate how extended objects (like strings) in $AdS_5 \times S^5$ would expand to $D3$ branes, just as the point-like gravitons do. A related issue has been addressed in [31]: in that paper, it was conjectured that nearly-BPS ultra-relativistic strings with large angular momenta $Q_I \gtrsim \sqrt{N}$ in S^5 would expand into giant gravitons. We show that a BPS cousin of this phenomenon, somewhat similar to the above, would appear in $\frac{1}{4}$-, $\frac{1}{8}$- and $\frac{1}{16}$-BPS sectors. We first show that there are limits where $D3$ giant gravitons shrink to string-like configurations. In some simple cases (to be explained below), the shinking D3 brane is tubular with topology $S^1 \times S^2$, where S^2 is small while S^1 remains macroscopic. The profiles of S^1 agree with those of the tensionless string solutions of [32] and generalizations thereof.

The organization of this paper is as follows. In section 2 we report our observation on the entropy of supersymmetric AdS_5 black holes. In section 3 we derive a class of $\frac{1}{16}$-BPS giant graviton solutions in $AdS_5 \times S^5$. In section 4 we present some examples. Especially, we point out the existence of tube-like solutions with tensionless string limits, analogous to the point-like graviton limit of spherical giant gravitons. We conclude in section 5 with several remarks. In the appendix, we generalize the tensionless string solutions of [32] to the $\frac{1}{16}$-BPS sector.

As we were preparing this paper, we recognized that section 3 overlaps with some results presented by Shiraz Minwalla in Strings 2006, Beijing. (See also [16].)

2. Entropy of supersymmetric AdS$_5$ black holes

Recently a large class of supersymmetric black hole solutions are discovered in 5 dimensional gauged supergravity [21 – 25], preserving 2 real supersymmetries. Although our main interest is the black holes in $AdS_5 \times S^5$, we start from more general supergravity theories, following [22]. We consider 5 dimensional $\mathcal{N}=1$ gauged supergravity coupled to n abelian vector multiplets. The bosonic fields are metric $g_{\mu\nu}$, one graviphoton plus $n-1$ vector fields collected together as A^I $(I = 1, \cdots, n)$, and $n-1$ real scalars ϕ^a $(a = 1, \cdots, n-1)$. One introduces n scalars $X^I(\phi^a)$ constrained as

$$\frac{1}{6} C_{IJK} X^I X^J X^K = 1\,, \tag{2.1}$$

where C_{IJK} are constants with symmetric IJK indices. We also define

$$X_I \equiv \frac{1}{6} C_{IJK} X^J X^K \quad \text{which satisfy} \quad X_I X^I = 1\ . \tag{2.2}$$

The bosonic part of this supergravity action is [33]

$$S = \frac{1}{16\pi G}\int \left(R^5 - 2\chi^2 \mathcal{V} - Q_{IJ} F^I \wedge *F^J - Q_{IJ} dX^I \wedge *dX^J - \frac{1}{6} C_{IJK} A^I \wedge F^J \wedge F^K \right)\,, \tag{2.3}$$

with the coupling matrix Q_{IJ}

$$Q_{IJ} = \frac{9}{2} X_I X_J - \frac{1}{2} C_{IJK} X^K \tag{2.4}$$

and the scalar potential $\mathcal{V}$

$$\mathcal{V} = \frac{9}{2} V_I V_J \left(Q^{IJ} - 2X^I X^J \right) , \tag{2.5}$$

where Q^{IJ} is the inverse matrix of Q_{IJ}. V_I is a constant vector, which is related to the vacuum value $\bar{X}_I$ of the scalars X_I,

$$\bar{X}_I = \xi^{-1} V_I \tag{2.6}$$

with a constant ξ. The latter constant is fixed by the vacuum value of the potential:

$$2\chi^2 \mathcal{V}(\bar{X}^I) \equiv -12\chi^2 \xi^2 \equiv -\frac{12}{\ell^2} . \tag{2.7}$$

The parameter ℓ is the radius of AdS_5 (supposing that $\mathcal{V}(\bar{X}^I)$ is not zero).

For the $\mathrm{U}(1)^3 \subset \mathrm{SO}(6)$ truncation of $\mathcal{N}=4$ gauged supergravity, which can be embedded into type IIB string theory in $AdS_5 \times S^5$ [34], one has three vector fields and

$$C_{123} = 1 \quad (\text{other} \;\; C_{IJK}\text{s are zero}) . \tag{2.8}$$

The three constrained scalars X^I, measuring the squashing of S^5, take vacuum values

$$\bar{X}^I = 1 \;\; (I = 1,2,3) \quad \rightarrow \quad \bar{X}_I = \frac{1}{3} . \tag{2.9}$$

When the scalars take these values, the internal S^5 becomes a round sphere. At any stage of our following analysis, inserting the above values will give us the results on black holes in $AdS_5 \times S^5$.

In the supergravity considered in [22, 25], the scalars ϕ^a live on a *symmetric space*, where the latter is specified by the following condition

$$C^{IJK} C_{J(LM} C_{PQ)K} = \frac{4}{3} \delta^I_{(L} C_{MPQ)} \tag{2.10}$$

with some C^{IJK}. For the S^5 case, this requirement is met by setting $C^{IJK} = C_{IJK}$ with (2.8). We briefly comment on other possible symmetric spaces in section 5.

We consider the black holes in the above supergravity theory. For simplicity, we first consider the black holes with self-dual angular momentum in AdS_5 [22], and comment on more general ones in [25] afterwards. Referring to [22] for the details, here we summarize the physical quantities. The $\mathrm{U}(1)^n$ charges Q_I are as follows (the normalization of the charges are commented below):

$$Q_I = \frac{\pi\ell}{G} \left(\frac{3}{4} q_I - \frac{3\alpha_2}{8\ell^2} \bar{X}_I + \frac{9}{8\ell^2} C_{IJK} \bar{X}^J C^{KLM} q_L q_M \right) , \tag{2.11}$$

where q_I $(I = 1, \cdots, n)$ are the n independent parameters of this solution. Defining

$$\alpha_1 \equiv \frac{27}{2} C^{IJK} \bar{X}_I \bar{X}_J q_K , \quad \alpha_2 \equiv \frac{27}{2} C^{IJK} \bar{X}_I q_J q_K , \quad \alpha_3 \equiv \frac{9}{2} C^{IJK} q_I q_J q_K , \tag{2.12}$$

the self-dual angular momentum is

$$J \equiv \frac{J_1 + J_2}{2} = \frac{\pi}{8G\ell}\left(\alpha_2 + \frac{2\alpha_3}{\ell^2}\right) \tag{2.13}$$

where J_1, J_2 are Cartans of the SO(4) rotation symmetry of AdS_5. The mass is given as

$$M = \frac{\pi}{4G}\left(\alpha_1 + \frac{3\alpha_2}{2\ell^2} + \frac{2\alpha_3}{\ell^4}\right) = \frac{1}{\ell}\left(\bar{X}^I Q_I + J_1 + J_2\ \right) \ . \tag{2.14}$$

$\bar{X}^I$ is the asymptotic value of the scalar X^I in this solution, the minima of the potential (2.5). The Bekenstein-Hawking entropy of the black hole is

$$S_{\rm BH} = \frac{A_{S^3}}{4G} = \frac{\pi^2}{2G}\sqrt{\alpha_3\left(1+\frac{\alpha_1}{\ell^2}\right) - \frac{\alpha_2^2}{4\ell^2}}\,, \tag{2.15}$$

given by the area of the horizon (squashed 3-sphere).

There are $n+1$ independent physical charges carried by this black hole: n electric charges Q_I's and the self-dual angular momentum $J = \frac{J_1+J_2}{2}$. However, there are only n independent parameters q_I of the solution. Thus, there is one relation between these charges. If one tries to express the macroscopic entropy in terms of physical charges Q_I and J, there should be an ambiguity in its expression [26, 27] as we mentioned in the introduction. We take advantage of this ambiguity and try to write (2.15) in terms of the physical charges in a simple way.

After some trials and errors, we find that the combination $C^{IJK}\bar{X}_I Q_J Q_K$ is interesting. Using (2.1), (2.2), (2.12) and the symmetric space condition (2.10), one obtains the following result after some algebra:

$$\frac{3}{2}C^{IJK}\bar{X}_I Q_J Q_K = \frac{1}{16}\left(\frac{\pi\ell}{G}\right)^2\left[\alpha_2 + \frac{3\alpha_3}{\ell^2} + \frac{\alpha_3\alpha_1}{\ell^4} - \frac{{\alpha_2}^2}{4\ell^4}\right] = \left(\frac{S_{\rm BH}}{2\pi}\right)^2 + \left(\frac{\pi\ell^3}{2G}\right) J\ . \tag{2.16}$$

Defining the constants

$$c \equiv \frac{\pi\ell^3}{8G}\ \ , \quad D^{IJ} \equiv \frac{1}{2}C^{IJK}(3\bar{X}_K)\,, \tag{2.17}$$

one obtains

$$S_{\rm BH} = 2\pi\sqrt{D^{IJ}Q_J Q_K - 4cJ}\ . \tag{2.18}$$

The constant c defined as (2.17) is actually the central charge of the holographically dual 4d superconformal field theory computed from the gravity data, normalized as $c = \frac{N^2}{4}$ for the $\mathcal{N}=4$ SU(N) Yang-Mills theory.

The convenient normalization convention of the electric charges Q_I depends on the way we embed this gauged supergravity into higher dimensional string or M-theories. Here we normalized them so that

$$\frac{1}{\ell}\bar{X}^I Q_I \tag{2.19}$$

becomes the electric charge contribution to the BPS mass (2.14). This normalization is natural in the $AdS_5 \times S^5$ case, since Q_I are internal Kaluza-Klein momenta associated with $\mathrm{U}(1)^3 \subset \mathrm{SO}(6)$ isometry, normalized as integers in our convention. In general, one may

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consider two kinds of electric charges. First, for those associated with internal isometries, the ℓ factor in front of (2.11) should be replaced by 'internal radii' in order to make them integral. In this case, the microscopic objects carrying these charges would be the (giant) gravitons. Second, if the internal manifold contains topological cycles, one can also have charged objects from wrapped branes. In this case, to define Q_I's as integral wrapping numbers, one needs to re-scale (2.11) such that $1/\ell$ in (2.19) is replaced by the volume times tension factor of the wrapped branes.

For the $AdS_5 \times S^5$ case, or the $\mathrm{U}(1)^3$ supergravity, the charges Q_I are integral as explained above. Inserting the values of the constants, (2.8) and (2.9), the Bekenstein-Hawking entropy (2.18) takes the form

$$S = 2\pi\sqrt{Q_1Q_2 + Q_2Q_3 + Q_3Q_1 - N^2 J}\ . \tag{2.20}$$

We again emphasize that the above expression is not unique from the black hole solutions we have, due to a relation of physical charges Q_I and J. We just discovered a simple way of writing it. For instance, [26] considered the 'small' black holes in the regime $Q_I \ll N^2$. The relation between the physical charges Q_I, J is

$$N^2 J \approx Q_1Q_2 + Q_2Q_3 + Q_3Q_1 - \frac{2}{N^2}Q_1Q_2Q_3 + O\left(\frac{Q_I}{N^2}\right)^4 \tag{2.21}$$

where all Q_I's are assumed to be of same order. (2.20) can be rewritten as

$$S \approx 2\sqrt{2}\pi\frac{\sqrt{Q_1Q_2Q_3}}{N} \ + \ \text{(higher order terms)}\ . \tag{2.22}$$

This form (2.22) was advocated by the authors of [26] to discuss the microscopic aspects. No matter what prescription one gives to this ambiguity, the *value* of the entropy is the same, of course, upon imposing the relation like (2.21). However, we think (2.20) would be significant since this 'simple' expression is exact in any regime of charges. It is likely that there would be a simple explanation of this degeneracy from the microscopic side, probably via a Cardy or Hardy-Ramanujan formula in a certain microscopic model. If this conjecture is true, it will be challenging to explain the $4c = N^2$ factors in (2.20).

From the expression (2.20), one finds that one of the three charges may be turned off, say $Q_3 = 0$, while having regular black holes. However, it is impossible to have black holes with single electric charge, say $Q_2 = Q_3 = 0$, since the quantity inside the square-root cannot be positive any more. This fact was also mentioned in [22]. Saying it differently, the self-dual angular momentum J of the black hole is bounded from above by the electric charges Q_I. From (2.20), or (2.18), the bound is

$$J_1 + J_2 \leq \frac{1}{2c}\, D^{IJ}Q_IQ_J \quad \text{ or } \quad J_1 + J_2 \leq \frac{2}{N^2}(Q_1Q_2 + Q_2Q_3 + Q_3Q_1)\ . \tag{2.23}$$

Existence of such an upper bound itself is familiar from the spinning black holes in the Minkowski space, perhaps except for the central charge factor N^2.

Finally, we turn to the black holes with general angular momenta $J_1 \neq J_2$. The black holes in [25] carry $n + 2$ physical charges Q_I, J_1 and J_2 and $n+1$ independent parameters

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in the solutions (generalizing q_I's above). There again is one relation between physical charges. Compared to the $J_1 = J_2$ solution explained in this section, there is one more parameter and physical charge, respectively. The way this additional parameter appears is rather involved, which is the reason why we do not present it here. Quite remarkably, we find that the simple expressions (2.18) and (2.20) continue to hold even in the case $J_1 \neq J_2$.

3. $\frac{1}{16}$-BPS giant gravitons

In this section we find $\frac{1}{16}$-BPS giant graviton solutions given by three holomorphic functions and discuss their properties.

The solution is given by the six complex coordinates Y_A $(A = 0, \cdots, 5)$ of $AdS_5 \times S^5 \subset \mathbb{R}^{2+4} \times \mathbb{R}^6$, which satisfy the constraints

$$|Y^0|^2 - |Y^1|^2 - |Y^2|^2 = 1 \quad \text{and} \quad |Y^3|^2 + |Y^4|^2 + |Y^5|^2 = 1\,. \tag{3.1}$$

With the orientation convention advocated in [14], Y^A's are decomposed as twelve 'x' and 'y' variables as[2]

$$\begin{aligned} &Y^0 = X^{-1} - iX^0,\ Y^1 = X^1 - iX^2,\ Y^2 = X^3 - iX^5, \\ &Y^3(\equiv Z^1) = X^5 + iX^6,\ Y^4(\equiv Z^2) = X^7 + iX^8,\ Y^5(\equiv Z^3) = X^9 + iX^{10}. \end{aligned} \tag{3.2}$$

In terms of 12 dimensional Dirac spinors (with 64 complex components), the $\frac{1}{16}$ supersymmetry condition is

$$\Gamma_A \Psi = 0 \quad (A = 1, 2, \cdots, 6) \tag{3.3}$$

with the above choice (3.2) of complex structure. The above 6 projectors define $\frac{1}{16}$ supersymmetry from 10 dimensional IIB viewpoint, since two of the above six are those reducing Ψ to IIB spinors [14]. The spinor Ψ satisfying this condition is the Clifford vacuum in 64 dimensional Hilbert space. $D3$ brane configurations preserving this supersymmetry should satisfy

$$\frac{1 - \Gamma^{\hat{r}_1 \hat{r}_2}}{2}(\Gamma - 1)\Psi = 0\,, \quad \Gamma = -\frac{i}{4!}\epsilon^{\mu\nu\rho\sigma}\gamma_{\mu\nu\rho\sigma} \tag{3.4}$$

where greek indices denote the components in local orthonormal frame basis of the world-volume $(\mu = 0, 1, 2, 3)$. The unit vectors $\hat{r}_1$ and $\hat{r}_2$ appearing in the superscripts are normals of AdS_5 and S^5, respectively. In complex coordinates, their components are (Y^0, Y^1, Y^2) and $(Y^3, Y^4, Y^5) \equiv (Z^1, Z^2, Z^3)$. With the condition (3.3), the supersymmetry requirement (3.4) is simplified as

$$\begin{aligned} \frac{1 - \Gamma^{\hat{r}_1 \hat{r}_2}}{2}\Bigg[-i\epsilon^{\mu\nu\rho\sigma}\bigg(&-\frac{1}{2}(t^A_\mu t^B_\nu t^{\bar{A}}_\rho t^{\bar{B}}_\sigma)\eta_{A\bar{A}}\eta_{B\bar{B}} + \frac{1}{2}(t^A_\mu t^{\bar{B}}_\nu t^{\bar{C}}_\rho t^{\bar{D}}_\sigma)\eta_{A\bar{B}}\Gamma_{\bar{C}\bar{D}} \\ &+\frac{1}{24}(t^{\bar{A}}_\mu t^{\bar{B}}_\nu t^{\bar{C}}_\rho t^{\bar{D}}_\sigma)\Gamma_{\bar{A}\bar{B}\bar{C}\bar{D}}\bigg) - 1\Bigg]\Psi = 0 \end{aligned} \tag{3.5}$$

[2]Under the U(1) transformations associated with the positive conserved charges M and $J_{1,2}$, one has $\delta Y^A = -iY^A$ for $A = 0, 1, 2$, while under those associate with $Q_{1,2,3}$, $\delta Y^A = iY^A$ for $A = 3, 4, 5$.

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where t_μ's are orthonormal tangent vectors on the worldvolume, push-forwarded to the bulk ($A, B = 1, \cdots, 6$):

$$\eta_{A\bar{B}}(t^A_\mu t^{\bar{B}}_\nu + t^A_\nu t^{\bar{B}}_\mu) = \eta_{\mu\nu} \ . \tag{3.6}$$

The second and third terms in (3.5) containing $\Gamma_{\bar{A}\bar{B}}\Psi$ and $\Gamma_{\bar{A}\bar{B}\bar{C}\bar{D}}\Psi$ are 'excited states' from the Clifford vacuum, which by themselves cannot cancel the first and last term proportional to Ψ. One should suitably choose the tangent vectors t^A_μ to make the second and third terms to be proportional to Ψ. One way is to let these unwanted terms vanish themselves. Another way would be to use the projector $\frac{1-\Gamma^{\hat{r}_1\hat{r}_2}}{2}$ in front, i.e., to have the unwanted terms be annihilated by this projector.

As for the first possibility, we require the worldvolume be given by a holomorphic hyperspace Σ_6 with complex dimension 3 in $\mathbb{C}^{1+2} \times \mathbb{C}^3$. This requirement is met by a zero locus of three holomorphic functions of Y^A. Intersecting it with $AdS_5 \times S^5$ would give the 4 dimensional worldvolume Σ_4. The tangent space of Σ_6 is closed under the action of complex structure, i.e., $I.v \subset T(\Sigma_6)$ if $v \subset T(\Sigma_6)$. Since we make two projections on tangent vectors associated with (3.1), two of the four tangent vectors in $T(\Sigma_4)$ should rotate into $\hat{r}_1$, $\hat{r}_2$ directions by the action of I. The remaining 2-plane is invariant under I: following [13], we call this subspace $T_0(\Sigma_4) \in T(\Sigma_4)$. Note that this subspace is spacelike since we projected out one of the two timelike directions in (3.1). We choose the two orthonormal bases of this subspace as t_z and $t_{\bar{z}} = (t_z)^*$ in complex basis, which satisfy

$$t^{\bar{A}}_z = 0 \ . \tag{3.7}$$

Let us also write other two unit vectors normal to this subspace as t_a ($a = 1, 2$). The orthonormality condition (3.6) simplifies as

$$\begin{aligned} \eta_{A\bar{B}} t^A_z t^{\bar{B}}_{\bar{z}} &= \eta_{z\bar{z}} = \frac{1}{2} \\ \eta_{A\bar{B}} t^A_z t^{\bar{B}}_a &= 0 \quad (a = 1, 2) \ . \end{aligned} \tag{3.8}$$

By requiring holomorphicity (3.7), the third term in (3.5) vanishes and one is left with

$$\begin{aligned} 0 &= \frac{1-\Gamma^{\hat{r}_1\hat{r}_2}}{2}\left[-i\epsilon^{z\bar{z}ab}\left((\eta_{A\bar{A}}t^A_z t^{\bar{A}}_{\bar{z}})(\eta_{B\bar{B}}t^B_a t^{\bar{B}}_b) + \frac{1}{2}(\eta_{A\bar{B}}t^A_z t^{\bar{B}}_{\bar{z}})(t^{\bar{C}}_a t^{\bar{D}}_b \Gamma_{\bar{C}\bar{D}})\right) - 1\right]\Psi \\ &= \frac{1-\Gamma^{\hat{r}_1\hat{r}_2}}{2}\left[-i\epsilon^{z\bar{z}ab}\left(\frac{1}{2}(\eta_{A\bar{B}}t^A_a t^{\bar{B}}_b) + \frac{1}{4}(t^{\bar{A}}_a \Gamma_{\bar{A}})(t^{\bar{B}}_b \Gamma_{\bar{B}})\right) - 1\right]\Psi \ . \end{aligned} \tag{3.9}$$

For the last equation to hold, one should impose some nice property to the remaining two vectors t_a. To guess what it can be, note that the projector $\frac{1-\Gamma^{\hat{r}_1\hat{r}_2}}{2}$ has the property

$$\frac{1-\Gamma^{\hat{r}_1\hat{r}_2}}{2}(\Gamma_{\hat{r}_1} - \Gamma_{\hat{r}_2}) = 0 \qquad (\Gamma_{\hat{r}_1} = -\Gamma^{\hat{r}_1}) \ . \tag{3.10}$$

Using (3.3), the matrix $\Gamma_{\hat{r}_1} - \Gamma_{\hat{r}_2}$ acted on Ψ is

$$(\Gamma_{\hat{r}_1} - \Gamma_{\hat{r}_2})\Psi = \left(\sum_{A=0,1,2} Y^{\bar{A}}\Gamma_{\bar{A}} - \sum_{A=3,4,5} Y^{\bar{A}}\Gamma_{\bar{A}}\right)\Psi \ . \tag{3.11}$$

To use the above properties, we require that the following null vector

$$t_+^A \equiv \frac{1}{2}(-iY^0, -iY^1, -iY^2, iY^3, iY^4, iY^5), \quad t_+^{\bar{A}} = (t_+^A)^* \tag{3.12}$$

be a tangent vector. With this vector, one can rewrite (3.11) as

$$(\Gamma_{\hat{r}_1} - \Gamma_{\hat{r}_2})\Psi = -2i(t_+^{\bar{A}}\Gamma_{\bar{A}})\Psi \quad , \quad (\Gamma_{\hat{r}_1} - \Gamma_{\hat{r}_2}) = +2i(t_+^A\Gamma_A) - 2i(t_+^{\bar{A}}\Gamma_{\bar{A}}) \ . \tag{3.13}$$

We also write the last tangent vector with the symbol t_-, which we also choose to be null. The two vectors are decomposed into a transverse time-evolution vector and a space-like tangent vector. We write $t_\pm = \gamma_\pm(\mathbf{e}_\psi \pm \mathbf{e}_\phi)$ with

$$\mathbf{e}_\phi = \cosh\rho\ (e_t, ve_\phi) \ , \quad \mathbf{e}_\psi = \cosh\rho\ \sqrt{1-v^2}(0, e_\psi) \ , \tag{3.14}$$

where e_ϕ and e_ψ are unit vectors in $\mathbb{R}^{10} \subset \mathbb{R}^{2+10}$ normal/tangent to the spatial configuration at given time, respectively: they are mutually orthogonal $e_\phi \cdot e_\psi = 0$. e_t is the unit time vector in $\mathbb{R}^2 \subset \mathbb{R}^{2+10}$, and v is the physical (=transverse) velocity. $\gamma_\pm$ are boost ambiguities in choosing the null local orthonormal frames preserving the canonical form of the metric

$$\eta_{+-} = \eta_{z\bar{z}} = \frac{1}{2} \ . \tag{3.15}$$

$\gamma_\pm$ are thus required to satisfy

$$\eta_{+-} = \frac{1}{2} = \eta_{A\bar{B}}(t_+^A t_-^{\bar{B}} + t_-^A t_+^{\bar{B}}) = 2(1-v^2)\gamma_+\gamma_- \cosh^2\rho \ . \tag{3.16}$$

Since our choice (3.12) already fixed $\gamma_+ = \frac{1}{2}$, we have

$$t_+ = \frac{1}{2}(\mathbf{e}_\psi + \mathbf{e}_\phi)\,, \quad t_- = \frac{1}{2(1-v^2)\cosh^2\rho}(\mathbf{e}_\psi - \mathbf{e}_\phi) \ . \tag{3.17}$$

Futhermore, the the normalization (3.15) requires

$$\epsilon^{z\bar{z}+-}\ \ (= (-2i)\cdot(-2)\epsilon^{xy\psi t}) = +4i \quad (\epsilon_{0123} = 1) \tag{3.18}$$

in the orientation convention of [14]. We note that

$$\eta_{A\bar{B}}(t_+^A t_-^{\bar{B}} - t_-^A t_+^{\bar{B}}) \sim (t_-, I.t_+) \sim (t_-, \hat{r}_1 - \hat{r}_2) = 0 \quad \to \quad \eta_{A\bar{B}}t_+^A t_-^{\bar{B}} = \eta_{A\bar{B}}t_-^A t_+^{\bar{B}} = \frac{1}{4}\,, \tag{3.19}$$

since t_- is orthogonal to both normal vectors $\hat{r}_1$ and $\hat{r}_2$. Having fixed all the tangent vectors, we completely specified our ansatz to solve the BPS equation.

With the above ansatz, the supersymmetry condition (3.9) becomes

$$\begin{aligned} 0 &= \frac{1-\Gamma^{\hat{r}_1\hat{r}_2}}{2}\left[-i\epsilon^{z\bar{z}+-}\left\{\frac{1}{2}\eta_{A\bar{B}}(t_+^A t_-^{\bar{B}} - t_-^A t_+^{\bar{B}}) - \frac{i}{4}(t_-^{\bar{A}}\Gamma_{\bar{A}})(\Gamma_{\hat{r}_1} - \Gamma_{\hat{r}_2})\right\} - 1\right]\Psi \\ &= \frac{1-\Gamma^{\hat{r}_1\hat{r}_2}}{2}\left[-i\epsilon^{z\bar{z}+-}(\eta_{A\bar{B}}t_+^A t_-^{\bar{B}}) - 1\right]\Psi = 0 \end{aligned} \tag{3.20}$$

using (3.18), (3.19). This completes the proof that any holomorphic hyperspace which includes $i(-Y^0, -Y^1, -Y^2, Y^3, Y^4, Y^5)$ as a tangent vector defines a worldvolume of $\frac{1}{16}$-BPS $D3$-brane. To summarize, the solution is given by three holomorphic 'weighted-homogeneous' functions

$$\begin{aligned} F_k(Y^0, Y^1, Y^2, Z^1, Z^2, Z^3) &= 0 \quad (k = 1, 2, 3)\,, \\ \text{where} \quad F_k(\lambda Y^A, \lambda^{-1} Z^A) &= \lambda^{d_k} F_k(Y^A, Z^A)\,. \end{aligned} \tag{3.21}$$

Note that $Z^A = Y^{A+2}$ for $A = 1, 2, 3$. The numbers d_k are suitable degrees of F_k. We stress that the $\pm$ weights of the coordinates Z^A and Y^A $(A = 1, 2, 3)$ is a consequence of (3.10), which in turn required (3.12) to be a tangent vector with the relative $-$ sign.

Now we show that the energy of this configuration saturates the BPS bound given by the sum of five $\mathrm{U}(1)^5 \in \mathrm{SO}(6) \times \mathrm{SO}(4)$ angular momenta. The energy conjugate to the time t (appearing in $Y^0 = \cosh\rho\, e^{-it}$) comes from two contributions. That coming from the DBI action reads

$$M_{\mathrm{DBI}} = N \int_{\Sigma_3} \cosh\rho\; vol(\Sigma_3) \frac{1}{\sqrt{1 - v^2}} \tag{3.22}$$

where Σ_3 denotes the spatial configuration of the brane in $AdS_5 \times S^5$ at given time, and N is the 5-form flux number. The volume of Σ_3 in the integrand is measured by the pull-back of the bulk metric. The energy also gets contribution from the Wess Zumino term if the brane is extended in AdS_5:

$$M_{\mathrm{WZ}} = -N \int_{\partial^{-1}\Sigma_3} 4 i(e^t).i(e^{\hat{r}_1}).\ \frac{1}{6} \tilde{\omega} \wedge \tilde{\omega} \wedge \tilde{\omega} \tag{3.23}$$

where

$$\omega = \frac{i}{2} \sum_{A=1}^{3} \eta_{A\bar{A}} dZ^A \wedge d\bar{Z}^{\bar{A}} \quad \text{and} \quad \tilde{\omega} = \frac{i}{2} \sum_{A,B=0}^{2} \eta_{A\bar{B}} dY^A \wedge d\bar{Y}^B \tag{3.24}$$

are the Kähler forms of $\mathbb{C}^3$ and $\mathbb{C}^{2+1}$ respectively, $e^{\hat{r}_2} \sim Z^A$ and $e^{\hat{r}_1} \sim Y^A$ (in components) are vectors normal to S^5 and AdS_5, respectively, and $e^t = (-iY^0, 0, 0)$ is the time translation vector. The sum of five angular momenta from the DBI action is

$$[Q_1 + Q_2 + Q_3 + J_1 + J_2]_{\mathrm{DBI}} = N \int_{\Sigma_3} \cosh\rho\; vol(\Sigma_3) \frac{v^2}{\sqrt{1 - v^2}}\,, \tag{3.25}$$

which comes from the fact that the diagonal of $\mathrm{U}(1)^5$ rotations is generated by the vector $\dot{\vec{X}} = (iZ^A, -iY^1, -iY^2)$, decomposed to transverse part $\cosh\rho\, \vec{v}$ and longitudinal one $\cosh\rho \sqrt{1 - v^2}\, e_\psi$. The 'sum of momenta minus the energy' from the Wess-Zumino term is computed in the same way as [13]:

$$\begin{aligned} \sum_{A=1^3} Q_A + J_1 + J_2 - M \bigg|_{\mathrm{WZ}} &= \frac{iN}{2} \int_{\Sigma_3} (Z \cdot d\bar{Z} - \bar{Z} \cdot dZ) \wedge \omega - (Y \cdot d\bar{Y} - \bar{Y} \cdot dY) \wedge \tilde{\omega} \\ &= N \int_{\Sigma_3} e^{\parallel} \wedge \omega - \tilde{e}^{\parallel} \wedge \tilde{\omega} \equiv N \int_{\Sigma_3} I.e^{\hat{r}_2} \wedge \omega - I.e^{\hat{r}_1} \wedge \tilde{\omega} \end{aligned} \tag{3.26}$$

where $(I.e^{\hat{r}_2}, I.e^{\hat{r}_1})$ in this integral denotes the 1-form dual to the real 10 dimensional vector (iZ^A, iY^1, iY^2), pull-backed to the world-volume. Note that the sign of the second term in the last integrand can be checked independently by requiring it be negative for $\frac{1}{2}$-BPS dual giant gravitons, for which $(0, -I.e^{\hat{r}_1})$ is the tangent vector. For the BPS energy to be given by the sum of five charges Q_A and $J_{1,2}$, we want (3.22) = (3.25) + (3.26), i.e.,

$$\int_{\Sigma_3} e^{\|} \wedge \omega - \tilde{e}^{\|} \wedge \tilde{\omega} \overset{!}{=} (3.22) - (3.25) = \int_{\Sigma_3} \cosh\rho \sqrt{1-v^2}\, vol(\Sigma_3) \equiv \int_{\Sigma_3} (e^{\|} - \tilde{e}^{\|}) \wedge (\omega + \tilde{\omega}) \,. \tag{3.27}$$

$e^{\|} - \tilde{e}^{\|}$ in the last integral is the 1-form dual to t_+ in (3.12). To see if this relation holds, one only needs to check

$$\int_{\Sigma_3} e^{\|} \wedge \tilde{\omega} - \tilde{e}^{\|} \wedge \omega = 0 \,. \tag{3.28}$$

Since the integrand of (3.28) is exact,

$$e^{\|} \wedge \tilde{\omega} - \tilde{e}^{\|} \wedge \omega \sim d\left(e^{\|} \wedge \tilde{e}^{\|}\right) \,, \tag{3.29}$$

the integral (3.28) is zero for *compact* D3 branes. Therfore, the energy is given as

$$M = Q_1 + Q_2 + Q_3 + J_1 + J_2 \,, \tag{3.30}$$

which obeys the same BPS relation as the black hole mass (2.14), taking into account that the latter is conjugate to ℓt in our coordinate.

4. Examples of giant gravitons

The geometry of the classical giant graviton solutions, given by holomorphic functions, is expected to be rich. One class which is relatively easy to examine is those whose world-volumes are nearly degenerate. A well-known degeneration is the spherical giant graviton shrinking to a point particle as angular momentum decreases. The $\frac{1}{2}$-BPS configuration

$$Z^3 Y^0 = \alpha \,, \quad Y^1 = Y^2 = 0 \tag{4.1}$$

degenerates to a point as $|\alpha|$ increases to 1.

In this section, as simple examples of the giant gravitons with nontrivial topology, we investigate various ways the $D3$ branes can shrink, where the 3+1 dimensional worldvolumes degenerate to 1+1 dimensional or 2+1 dimensional objects.[3] The nearly-degenerate configurations are easy to analyze, and we show that the world-volumes of the $D3$ branes we treat in this section are topologically tubular: $S^1 \times S^2$ or $S^1 \times S^1 \times S^1$. The cross sections S^2 or $S^1 \times S^1$ of the tubes are small in our examples, just like small S^3 for nearly point-like gravitons. One obvious interpretation of these objects would be superpositions of point-like gravitons, which may also be supported by [16] in the $\frac{1}{8}$-BPS sector. We would also like to give them an interpretation in terms of tensionless strings.

[3] For $\frac{1}{8}$-BPS giant gravitons, there is another limit where the geometry is easy to analyze. It is the stationary giant gravitons in S^5 given by a single homogeneous function. They are given by circle fibrations over complex algebraic curves.

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4.1 $\frac{1}{4}$- and $\frac{1}{8}$-BPS examples

We first investigate the giant gravitons in $\frac{1}{4}$- and $\frac{1}{8}$-BPS sectors, which were treated by Mikhailov. For simplicity, let us first consider the following $\frac{1}{4}$-BPS solution

$$Z^1 Z^2 (Y^0)^2 = \alpha \ , \quad Y^1 = Y^2 = 0 \ . \tag{4.2}$$

This actually belongs to a class of $\frac{1}{8}$-BPS solutions discussed in [13],

$$(Z^1)^{m_1}(Z^2)^{m_2}(Z^3)^{m_3}(Y^0)^{m_1+m_2+m_3} = \alpha \, , \quad Y^1 = Y^2 = 0 \tag{4.3}$$

with integral m_A. The three S^5 momenta Q_A are proportional to the vector (m_1, m_2, m_3), so (4.2) carries $Q_1 = Q_2$ and $Q_3 = 0$. We would like to study the geometry of this brane as one increases α, which one may simply choose to be a positive number. Since we are interested in the intersection of (4.2) with S^5, the variables Z^1 and Z^2 are bounded from above. This means that the configuration given by the equation (4.2) will not intersect S^5 any more if α is too large. The maximal value to have nonempty intersection is $\alpha = \frac{1}{2}$, for which one obtains

$$Z^3 = 0 \, , \quad Z^1 = \frac{1}{\sqrt{2}} \, e^{i\phi} \, , \quad Z^2 = \frac{1}{\sqrt{2}} \, e^{-i\phi} \ . \tag{4.4}$$

As for the phases, the equation (4.2) requires $\phi_1 = -\phi_2 (\equiv \phi)$. One can see that this configuration degenerates to a circular string in S^5 parameterized by the angle ϕ, located at $|Z^1| = |Z^2| = \frac{1}{\sqrt{2}}$ (and $Z^3 = 0$). To see how the $D3$ brane degenerates, one may check a slightly resolved configuration $\alpha = \frac{1}{2} - \epsilon^2$, where $\epsilon \ll 1$ is a small positive number. With the following parametrization

$$Z^1 = \sin\Theta \cos\theta \ e^{i\phi^1} \ , \quad Z^2 = \sin\Theta \sin\theta \ e^{i\phi^2} \ , \quad Z^3 = \cos\Theta \ e^{i\phi^3} \tag{4.5}$$

of S^5, one obtains

$$\frac{1}{2}\left(\Theta - \frac{\pi}{2}\right)^2 + \left(\theta - \frac{\pi}{4}\right)^2 \approx \epsilon^2 \, , \quad \phi^1 = -\phi^2 \ (\equiv \phi) \ . \tag{4.6}$$

The angle ϕ^3 is a tangent direction of the brane. The variables Θ, ϕ^3 and θ form an ellipsoid whose size is ϵ, while the angle ϕ parameterizes the macroscopic circle of the string. Therefore, the configuration is topologically a thin tube with S^2 cross section.

The time evolution of this nearly degenerate 'string' is given by $\dot{\phi}^1 = \dot{\phi}^2 = 1$, with a lightlike transverse velocity. Since the motion is nearly lightlike for small ϵ, the energy and the angular momenta are dominantly given by the DBI action for nearly-degenerated branes. To the leading order in ϵ, they are given as

$$Q_1 = Q_2 \approx \frac{N}{4\pi^2} \int_{S^1 \times S^2} \frac{1}{\sqrt{1 - v^2}} \approx 2N\epsilon \tag{4.7}$$

while $Q_3 = 0$ exactly [13]. We used the following expression for the transverse speed,

$$v^2 = \frac{4\alpha^2}{|Z^1|^2 + |Z^2|^2} = (1 - 2\epsilon^2)\sin(2\theta) \quad \rightarrow \quad 1 - v^2 \approx 2\epsilon^2 + 2\left(\theta - \frac{\pi}{4}\right)^2 \tag{4.8}$$

up to $O(\epsilon)^4$ corrections that we can ignore.

As mentioned in the beginning of this section, the above configuration may be interpreted as a collection of nearly point-like giant gravitons: at least in the degenerate limit, it looks like a superposition of super-gravitons. The way this object expands in the above solution, as angular momenta $Q_1 = Q_2$ increase, is a single tube with S^2 cross section. The above configuration has been also treated in the literature as a tensionless string solution [32]. The analysis of the tensionless strings is reviewed in the appendix, generalizing [32] to the $\frac{1}{16}$-BPS sector. The shape of the $\frac{1}{4}$-BPS solution with $Q_1 = Q_2$ agrees with what we presented above with giant gravitons.[4]

$\frac{1}{4}$-BPS tensionless strings with $Q_1 \neq Q_2$ can also be reproduced from the degeneration of giant gravitons. It is simply given by the equation

$$(Z^1)^{m_1}(Z^2)^{m_2} = \alpha \quad \text{with} \quad \alpha \to \sqrt{\frac{(m_1)^{m_1}(m_2)^{m_2}}{(m_1+m_2)^{m_1+m_2}}} \; . \tag{4.9}$$

The ratio of charges are given as $(Q_1, Q_2) \propto (m_1, m_2)$, whose value is determined by the α parameter as in the previous example. The string is located in the limit at

$$Z^3 = 0\,, \quad |Z^1| = \sqrt{\frac{m_1}{m_1+m_2}}\,, \quad |Z^2| = \sqrt{\frac{m_2}{m_1+m_2}} \tag{4.10}$$

while satisfying $m_1\phi^1 + m_2\phi^2 = 0$. The string is parameterized by an angle σ with range $0 \sim 2\pi$, which is related to $\phi^{1,2}$ by $\phi^1 = m_2\sigma$, $\phi^2 = -m_1\sigma$. The shape and the charges again agree with the tensionless string solutions. From the result derived in our appendix, the agreement is clear if one sets $Q_3 = J_1 = J_2 = 0$.

One can also investigate (nearly) degenerate configurations in the $\frac{1}{8}$-BPS sector with (4.3). Near the maximal value $\alpha = \sqrt{\frac{(m_1)^{m_1}(m_2)^{m_2}(m_3)^{m_3}}{(m_1+m_2+m_3)^{m_1+m_2+m_3}}}$, the topology of the $D3$ brane is $S^1 \times S^1 \times S^1$ where the last S^1 shrinks. It is located at

$$|Z^1| = \sqrt{\frac{m_1}{m_1+m_2+m_3}}\,, \quad |Z^2| = \sqrt{\frac{m_2}{m_1+m_2+m_3}}\,, \quad |Z^3| = \sqrt{\frac{m_3}{m_1+m_2+m_3}} \tag{4.11}$$

while macroscopic $S^1 \times S^1$ is spanned by three angles ϕ^A subject to a constraint

$$m_1\phi^1 + m_2\phi^2 + m_3\phi^3 = 0 \; . \tag{4.12}$$

In comparison, the tensionless strings in [32] with charges $Q_A \propto m_A$ are also located at (4.11) and stretched along a line in the above $S^1 \times S^1$ described by (4.12). The $S^1 \times S^1$ ($\times$ small S^1) may be interpreted as a collection of such tensionless strings.

[4]Originally, these tensionless strings were regarded as an ultra-relativistic limit of the nearly-BPS Nambu-Goto strings in $AdS_5 \times S^5$ [31, 32]. The above giant graviton is not directly related to the latter. As a BPS cousin of the fundamental strings, one may put (local) fundamental string charges to the above tubular brane by turning on electric flux along the string direction, following [14].

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4.2 $\frac{1}{16}$-BPS examples

Finally we present similar $\frac{1}{16}$-BPS degenerate objects. We will also see that the $\pm$ weights in (3.21) play some roles for such configurations to exist. For simplicity we only consider $\frac{1}{16}$-BPS examples with $Q_3 = 0$.[5] We take the following holomorphic functions:

$$\begin{aligned} \alpha &= (Z^1)^{m_1}(Z^2)^{m_2}(Y^0)^{m_1+m_2} \,, \\ \frac{Y^1}{Y^0} &= \beta_1(Z^1)^{p_1}(Z^2)^{p_2}(Y^0)^{p_1+p_2} \\ \frac{Y^2}{Y^0} &= \beta_2(Z^1)^{q_1}(Z^2)^{q_2}(Y^0)^{q_1+q_2} \,. \end{aligned} \tag{4.13}$$

$\vec{p}$ and $\vec{q}$ are rational, while $\vec{m}$ is integral. We use the the following coordinates of AdS_5

$$Y^0 = \cosh\rho\, e^{-it}\,, \quad Y^1 = \sinh\rho\ \cos\vartheta\ e^{-i\varphi^1}\,, \quad Y^2 = \sinh\rho\ \sin\vartheta\ e^{-i\varphi^2}\,, \tag{4.14}$$

while the S^5 coordinates are same as (4.5). At $t = 0$, the four angles $\phi^{1,2}$ and $\varphi^{1,2}$ are constrained as

$$\vec{m}\cdot\vec{\phi} = 0\ , \quad \varphi^1 = -\vec{p}\cdot\vec{\phi}, \quad \varphi^2 = -\vec{q}\cdot\vec{\phi}\,. \tag{4.15}$$

Introducing an angle σ (with range $0\sim 2\pi$), one can write the above four angles as

$$\begin{aligned} (\phi^1,\phi^2) &= (t + m_2\sigma,\ t - m_1\sigma) \equiv t(1,1) + \vec{r}\sigma \\ (\varphi^1,\varphi^2) &= \Big(t + (m_1p_2 - m_2p_1)\sigma,\ t + (m_1q_2 - m_2q_1)\sigma\Big) \equiv t(1,1) + \vec{s}\sigma \end{aligned} \tag{4.16}$$

for general t. Since the angle ϕ^3 is free, there are two independent angle-like directions tangent to the brane. One combination of other four variables (Θ, θ, ρ, φ) parameterizing $|Z^A|$ and $|Y^A|$ forms the last tangent direction. We first show that the brane can degenerate along this last direction. The degeneration appears at $|Z^3| = 0$, which makes the ϕ^3 direction degenerate also. The resulting configuration would therefore be string-like, which will be identified with the $\frac{1}{16}$-BPS tensionless strings treated in the appendix.

On the holomorphic hyperspace Σ_6 in $\mathbb{C}^{1+5}$ given by (4.13), three of the six tangent vectors lie in the 6 dimensional subspace $\mathbb{R}^{1+5}$ of $\mathbb{C}^{1+5}$ generated by $|Z^A|$ and $|Y^A|$: by intersecting it with $AdS_5 \times S^5$, one obtains a 1 dimensional locus which we claim to degenerate. The three normals (1-forms) to Σ_6 in $\mathbb{R}^{5+1}$, in ($|Z^A|$, $|Y^A|$) components, are

$$\begin{aligned} \mathbf{a} &\equiv \left(\frac{m_1}{|Z^1|},\ \frac{m_2}{|Z^2|},\ 0,\ \frac{m_1+m_2}{|Y^0|},\ 0,\ 0\right) \\ \mathbf{b} &\equiv \left(\frac{p_1}{|Z^1|},\ \frac{p_2}{|Z^2|},\ 0,\ \frac{p_1+p_2+1}{|Y^0|},\ -\frac{1}{|Y^1|},\ 0\right) \\ \mathbf{c} &\equiv \left(\frac{q_1}{|Z^1|},\ \frac{q_2}{|Z^2|},\ 0,\ \frac{q_1+q_2+1}{|Y^0|},\ 0,\ -\frac{1}{|Y^2|}\right)\ . \end{aligned} \tag{4.17}$$

These are gradients of (4.13). Intersecting the hyperspace with $AdS_5 \times S^5$, the single tangent vector of Σ_4 in $\mathbb{R}^{5+1}$ is orthogonal to the normals (4.17) as well as

$$e^{\hat{r}_2} \equiv (|Z^1|, |Z^2|, |Z^3|, 0, 0, 0) \quad \text{and} \quad e^{\hat{r}_1} \equiv (0, 0, 0, |Y^0|, -|Y^1|, -|Y^2|)\,, \tag{4.18}$$

[5]General $\frac{1}{16}$-BPS examples we find are similar to the $\frac{1}{8}$-BPS ones, degenerating to $2+1$ dimensions.

the normals of $AdS_5\times S^5$. When the (claimed) degeneration appears, the rank of these five normals should reduce so that the 'tangent' vector orthogonal to them would be ambiguous. In other words, one has a linear relation between the above five normals,

$$\mathbf{a}+\lambda_1\mathbf{b}+\lambda_2\mathbf{c}=\mu e^{\hat{r}_2}+\nu e^{\hat{r}_1} \tag{4.19}$$

with suitable coefficients. We rewrite them in components as

$$\text{1st and 2nd}:\ \big(\,|Z^1|^2,\ |Z^2|^2\,\big)=\frac{\vec{m}+\lambda_1\vec{p}+\lambda_2\vec{q}}{\mu} \tag{4.20}$$

$$\text{3rd}:\ |Z^3|=0 \tag{4.21}$$

$$\text{4th}:\ 1=\frac{m_1+m_2}{\nu}+(p_1+p_2)|Y^1|^2+(q_1+q_2)|Y^2|^2 \tag{4.22}$$

$$\text{5th and 6th}:\ \big(\,|Y^1|^2,\ |Y^2|^2\,\big)=\frac{\vec{\lambda}}{\nu}\,. \tag{4.23}$$

Note that, applying the requirement $|Z^1|^2+|Z^2|^2+|Z^3|^2=1$ to (4.20) and (4.21), one obtains

$$\frac{\mu}{\nu}=\frac{m_1+m_2}{\nu}+(p_1+p_2)|Y^1|^2+(q_1+q_2)|Y^2|^2\,. \tag{4.24}$$

Comparing it with (4.22), one obtains $\mu=\nu$. Here, the fact that the weights of Z^A and Y^A are opposite in counting homogeneity of the holomorphic functions (3.21) is crucial for the degenerate configurations to exist.[6]

The charges of the nearly degenerated branes are again dominantly given by DBI contributions,

$$Q_A\approx\int_{\Sigma_3}vol(\Sigma_3)\frac{|Z^A|^2}{\sqrt{1-v^2}}\,,\quad J_a\approx\int_{\Sigma_3}vol(\Sigma_3)\frac{|Y^a|^2}{\sqrt{1-v^2}} \tag{4.25}$$

where $v^2\approx 1$ $(A=1,2,3,\ a=1,2)$. For the above configurations, where $|Z^A|$ and $|Y^a|$ are nearly constants, the charges are proportional to

$$(\vec{Q},\ \vec{J})\propto(|Z^A|^2,\ |Y^a|^2)\,. \tag{4.26}$$

From this structure, one can reproduce (A.3), (A.4) and (A.5) for the tensionless strings. Also, note that the charges $(\vec{Q},\vec{J})$ are orthogonal to the tangent direction generated by ϕ^3 and σ translations in (4.16). The argument is the same as that in [13] for (4.3). As the result, one obtains

$$\vec{r}\cdot\vec{Q}+\vec{s}\cdot\vec{J}=0\,,\quad Q_3=0\,, \tag{4.27}$$

which is just (A.9) in the $Q_3=0$ case.

[6] For instance, consider a hyperspace given by genuine homogeneous functions, where Y^0 factors in the right hand sides of (4.13) are inverted. Then the terms with $|Y^1|^2$ and $|Y^2|^2$ in (4.22) would acquire additional $-$'s, incompatible with (4.20) unless $|Y^a|^2=0$.

5. Concluding remarks

In this paper we explored the $\frac{1}{16}$-BPS objects of type IIB string theory in $AdS_5 \times S^5$. First, in the gravity side, we investigated the supersymmetric black holes in gauged supergravity and pointed out that there is a simple expression for the Bekinstein-Hawking entropy in terms of physical charges. We think the simplicity of the expression suggests the existence of its nice microscopic explanation. In the microscopic side, we investigated the classical solutions of $\frac{1}{16}$-BPS giant gravitons. We obtained a class of solutions given by three holomorphic functions which are homogeneous with suitable weights on the bulk coordinates. As examples, we studied string-like degenerations of these solutions.

An obvious work to be done in the microscopic side would be to see whether there are more general $\frac{1}{16}$-BPS solutions than (3.21). Also, one may hope to obtain clues of understanding the black hole microphysics from the solution (3.21) or its generalizations. Especially, it will be very interesting to see whether there are microscopic configurations saturating the angular momentum bound (2.23). An analogous issue has been addressed for the rotating objects in Minkowski space. For instance, the entropy of rotating $D1$-$D5$ on T^5 (or $K3$) is $S = 2\sqrt{2}\pi\sqrt{Q_1Q_5 - J}$ (or $4\pi\sqrt{Q_1Q_5 - J}$). Here the zero entropy configurations are given by the circular supertubes [35 – 38], or U-duals of them [39 – 42]. With the AdS_5 black holes, it may be interesting to consider the 2-charge black holes with $S = 2\pi\sqrt{Q_1Q_2 - N^2J}$, even if the N^2 factor makes the problem more challenging.

One interesting feature of the $\frac{1}{16}$-BPS solutions discovered in this paper is that, they may be viewed as $\frac{1}{16}$-BPS deformations of either $\frac{1}{8}$-BPS giant gravitons of [13], or the $\frac{1}{8}$-BPS dual giant gravitons of [17]. For instance, the three holomorphic equations can be arranged in one of the two forms

$$F(Z^AY^0) = 0\,, \quad \frac{Y^a}{Y^0} = f^a(Z^AY^0) \quad (a = 1,2) \tag{5.1}$$

$$Z^AY^0 = F^A\left(\frac{Y^1}{Y^0}, \frac{Y^2}{Y^0}\right) \quad (A = 1,2,3)\,, \tag{5.2}$$

at least locally. (5.1) is a deformation of [13], the latter being given by $f^a \equiv 0$. (5.2) may be viewed as a deformation of dual giant gravitons discussed in [17], given by $F^A = c^A$ where c^A's are constants. It will be interesting to further explore these dual descriptions, since they seem to be amalgamated into a single solution space in the $\frac{1}{16}$-BPS sector.

As for our work on AdS_5 black holes, we stress that the Bekenstein-Hawking entropy of AdS_5 black holes admit simple expressions not only in the $\mathrm{U}(1)^3$ supergravity (truncating $\mathrm{SO}(6)$ gauged supergravity), obtained by an S^5 reduction of type IIB string theory. The only requirement for the supergravity is that the scalars in the vector multiplet live on a symmetric space. For instance, this is accomplished by AdS_5 supergravity with no less than 16 supersymmetries [43]. For instance, one can obtain $\mathcal{N} = 2$ AdS_5 supergravities by compactifying M-theory on suitable 6-manifolds. An explicit example, known as the $\mathcal{N} = 2$ Maldacena-Nunez solution [44], falls into this class. It will be interesting to consider black holes in this background, carrying electric charges coming from the membrane wrapping an internal 2-cycle, as well as those coming from internal momenta (analogous to the S^5

Cartans). In this case, the central charge factor c in front of the angular momentum J is proportional to N^3, where N is the number of $M5$ branes on which the dual superconformal field theory lives.

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A. The $\frac{1}{16}$-BPS tensionless strings

In this appendix we obtain the tensionless string solutions that are $\frac{1}{16}$-BPS, which have BPS energy $M = Q_1 + Q_2 + Q_3 + J_1 + J_2$. The bosinic action for the tensionless string is, generalizing that of [32],

$$\begin{aligned}\mathcal{L} &= p_\rho \dot{\rho} + p_\vartheta \dot{\vartheta} + \vec{J}\cdot\dot{\vec{\varphi}} + p_\theta \dot{\theta} + p_\psi \dot{\psi} + +\vec{Q}\cdot\dot{\vec{\phi}} \\ &\quad - \cosh\rho \sqrt{{p_\rho}^2 + \frac{{p_\vartheta}^2}{\sinh^2\rho} + \frac{1}{\sinh^2\rho}\left(\frac{{J_1}^2}{\cos^2\vartheta} + \frac{{J_2}^2}{\sin^2\vartheta}\right) + {p_\theta}^2 + \frac{{p_\psi}^2}{\sin^2\theta} + \sum_{A=1}^{3}\frac{Q_A^2}{n_A^2}} \\ &\quad -\lambda\left(p_\rho\rho' + p_\vartheta\vartheta' + \vec{J}\cdot\vec{\varphi}' + p_\theta\theta' + p_\psi\psi' + +\vec{Q}\cdot\vec{\phi}'\right) \end{aligned} \tag{A.1}$$

where ρ, ϑ and $\varphi_{1,2}$ parameterize the spatial directions of global AdS_5, and Θ, θ, ψ and $\phi^{1,2,3}$ parameterize the S^5 with the following metric

$$ds_{S^5}^2 = d\theta^2 + \sin^2\theta d\psi^2 + \sum_{A=1}^{3} n_A^2 (d\phi^A)^2 \ . \tag{A.2}$$

The coefficients n_A parameterize a unit S^2: $(n_A) = (\cos\theta, \sin\theta\cos\psi, \sin\theta\sin\psi)$, which are $|Z^A|^2$ in our previous notation. The term in the second line of (A.1) is minus the Hamoiltonian $\mathcal{H}$, and the last line with Lagrange multiplier λ is inserted for the string reparametrization constraint. The prime denotes the differentiation with the spatial coordinate of the worldsheet, say $\sigma(\sim \sigma + 2\pi)$.

Generalizing [32], the solution we are interested in has constant ρ, ϑ in AdS_5 and constant n_A's in S^5, together with $p_\rho = p_\vartheta = p_\theta = p_\psi = 0$. Other five momena $\vec{Q}$, $\vec{J}$ as well as the energy $\mathcal{H}$ are constants of motion when integrated with σ. We are interested in determining the values of these constant coordinates in terms of the constants of motion.

We start from the equation of motion coming from δn_A. Using $p_\theta = p_\psi = 0$, the equation of motion requires

$$n_A^2 = \frac{Q_A}{\sum_B Q_B} \ . \tag{A.3}$$

Similarly, $\delta\vartheta$ equation yields

$$(\cos^2\vartheta, \sin^2\vartheta) = \left(\frac{J_1}{J_1+J_2}\,,\ \frac{J_2}{J_1+J_2}\right)\,, \tag{A.4}$$

while $\delta\rho$ equation reduces to

$$\tanh^2\rho = \frac{J_1+J_2}{\mathcal{H}} \quad \left(\rightarrow \quad \cosh^2\rho = \frac{\mathcal{H}}{\mathcal{H}-J_1-J_2}\right)\ . \tag{A.5}$$

Inserting these values back into the expression of the Hamiltonian (second line of (A.1)), one obtains

$$\mathcal{H} = \sum_A Q_A + J_1 + J_2\,, \tag{A.6}$$

which is the BPS energy relation that matches with the $\frac{1}{16}$-BPS giant gravitons. Now we turn to the equations of motion for the momentum variables. Those coming from δp_ρ, δp_ϑ, δp_θ and δp_ψ turn out to be trivially satisfied. The equations of motion for $\delta\vec{Q}$ and $\delta\vec{J}$ reduce to

$$\dot{\vec{\phi}} = 1 + \lambda\vec{\phi}' \ \ , \quad \dot{\vec{\varphi}} = 1 + \lambda\vec{\varphi}'\ . \tag{A.7}$$

As in [32], we choose the gauge $\lambda = 0$. This results in a light-like time evolution for this string, which is

$$\phi^A = t + r^A\sigma \ \ , \quad \varphi^a = t + s^a\sigma \ \ . \tag{A.8}$$

As for the σ dependent terms, we chose the string to uniformly wrap the angle directions, as in [32]. Finally, one has to impose the constraint coming from $\delta\lambda$. With the above solution (A.8) for the angles, the constraint is

$$\vec{r}\cdot\vec{Q} + \vec{s}\cdot\vec{J} = 0\ . \tag{A.9}$$

This completes the construction of $\frac{1}{16}$-BPS tensionless string solutions.

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